A STUDY OF HEAT TRANSFER BETWEEN A FLAT CALORIMETER AND A PARTIALLY PACKED FLUIDIZATION BED OF LARGE HEIGHT

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532.546.2

Experiments were performed in an apparatus 4 m high and 0.6 × 1.2 m in the plan view, with gas distributor caps. The bed was 1.5 m above the outlet orifices in the distributor caps; it was made of electrical-grade corundum grains with an average equivalent diameter of 320 μm. The tests were ran in a free bed as well as in a bed packed with two sets of vertical tubes, both separated by a gap 0.16 m wide for accommodating a flat calorimeter 0.48 × 0.82 (height) × 0.03 m. This calorimeter consisted of five copper plates 0.45 × 0.15 (height) × 0.005 m on top of uniformly spaced heaters. The coefficients of heat transfer between a plate and the bed were determined from the measurements of the temperature difference between them and of the amount of heat input.

The result of using a partially packed bed was, first of all, a much lower heat transfer coefficient and, secondly, a not necessarily uniform heat transfer between calorimeter and bed, the authors supplied air additionally below the bottom edge of the calorimeter through a pipe with two lengthwise slots, each 450 mm long and 3 mm wide, which was located 100 mm above the outlet orifices in the distributor caps.

With additional air supplied underneath the calorimeter, the stirring rate of material in the gaps between the calorimeter and the packing tubes increased, while the material was vibrating predominantly up and down the calorimeter plane. The material was strongly stirred by bubbles rising along the calorimeter surface, within the region between the calorimeter and the packing, so that cold particles from the bed core acquired an easy access to the calorimeter plates. As a consequence, the heat transfer processes improved.

The mean heat transfer coefficients along the calorimeter height in a free fluidization bed increased somewhat under a higher relative rate of air flow through the auxiliary pipe, as long as the fluidization rate was moderate, and remained almost constant at high fluidization rates, i.e., when the heat transfer coefficient became high even without additional air supply. The heat transfer became then more uniform along the calorimeter height even at high fluidization rates.

In a partially packed bed, therefore, an increase in the relative rate of air flow resulted in a higher mean heat transfer coefficient at all fluidization rates within the test range. With a more uniform heat transfer along the calorimeter in a partially packed bed, therefore, heat transfer coefficients equal to those in a free fluidization bed are attainable.

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HEAT AND MASS TRANSFER DURING EVAPORATION OF MIXTURES IN A FILM COLUMN

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UDC 66.048

In order to solve problems in mass transfer during evaporation, it is necessary to consider also the heat transfer processes which occur concurrently as well as nonequimolar mass transfer and the effect of diffusive interaction between mixture components.

Adding these phenomena to the description of the entire process yields the following system of equations:

$$\frac{dy_i}{dF} = \frac{y_0}{G_0}\left[\sum_j \beta_{ij}^{y}\,(y_{if}y_j - y_i y_{jf}) + \frac{dG}{dF}\,\frac{y_{if} - y_i}{2}\right], \tag{1}$$

$$\frac{dy_0}{dF} = \frac{2y_0^2 \sum\limits_j \beta_{0j}\,(y_{0f}\,y_j - y_0 y_{jf})}{G_0\,(y_0 - y_{0f})}, \tag{2}$$

$$\frac{d\tau}{dF} = \frac{\sum\limits_i (H_{if} - H_i)\,\frac{dG_i}{dF} - \alpha^{y}\,(\tau - t_f)}{G\sum\limits_i y_i\,\frac{\partial H_i}{\partial \tau}}, \tag{3}$$

$$\frac{dt}{dF} = \frac{\sum\limits_i (H_{if} - h_i)\,\frac{dG_i}{dF} - \alpha^{y}\,(\tau - t_f) - q_{\mathrm{L}}}{L\sum\limits_i x_i\,\frac{\partial h_i}{\partial x_i}}, \tag{4}$$

$$\alpha^{y}\,(\tau - t_f) - \alpha^{x}\,(t_f - t) - \sum_i (H_{if} - h_{if})\,\frac{dG_i}{dF} = 0, \tag{5}$$

$$x_i = \frac{Gy_i + L_{\text{н}}x_{i\text{н}} - G_{\text{н}}y_{i\text{н}}}{G + L_{\text{н}} - G_{\text{н}}}, \tag{6}$$

$$\beta^{x}\,(x_i - x_{if}) + \frac{dG}{dF}\,\frac{x_i + x_{if}}{2} - \frac{dG_i}{dF} = 0, \tag{7}$$

with the molar flow rates of gas and liquid G and L respectively, the molar fractions of gas and liquid x and y respectively, the interface area F, the coefficients of heat and mass transfer α and β respectively, the heat content in gas and in liquid H and h respectively, the temperatures of liquid and gas τ and t respectively, the heat loss q_L, with the subscripts f referring to the interphase boundary, l referring to the bottom section of a column, and i, j referring to the mixture components.

A comparison between calculated and measured data pertaining to the evaporation of an n-hexane and n-heptane mixture into air in a film column have confirmed the validity of these equations and the feasibility of using system (1)-(7) for an analysis of this process. Such an analysis has shown that a large net flow resulting from evaporation causes the second component in the liquid phase to be transferred in the direction opposite to the direction of the conventional mass motive force. The result of nonequimolar transfer in the gaseous phase is that the effective mass transfer coefficient becomes the largest of all β_{ij} coefficients and also larger than predicted by theory of measured in an evaporation experiment with pure components.

The interplay between processes of heat and mass transfer produces a dip in the temperature profile of the liquid along the column height (at equal entrance and exit temperatures).

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A STUDY OF THE VORTEX EFFECT IN STEAM

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UDC 621.181.8

A series of geometrically similar vortex tubes operating with superheated steam under various sets of conditions was the subject of an experimental study to establish the feasibility of generalizing the test results in universal form.

The results of this study indicate that:

1. The value of η_h for a vortex tube operating with steam is 15-30% lower than for one operating with air at the same expansion ratio $\pi_c = P_1^0/P_c^0$ at the cold end.

2. With a smaller pressure reduction π_h at the hot end, the value of η_h remains almost constant over the given range of π_h.

3. As the vortex tube is scaled up dimensionally, its performance becomes more efficient.

4. As the pressure P_1^0 is raised by a factor of 2-3, with π_h unchanged, η_h also remains the same regardless of the diameter D. Moreover, π_c remains constant too (at a constant μ) and, consequently, the operating modes at various pressure levels may be regarded as being similar.

5. There is no change in η_h as the absolute temperature of the incoming steam is raised by 15-20%.

The question arises now whether the test results pertaining to vortex tubes may be represented in the form of a universal dimensionless relation between criterial simplex and complex groups, such groups being usually derived from the condition of identicity at the boundaries.

In an endeavor to represent the Reynolds number as a complex of inlet parameters, one can (under conditions of a supercritical head in the discharge nozzle, usual for vortex tubes) write

$$\mathrm{Re} \sim \frac{DP_1^0}{\sqrt{RT_1^0}\,\mu'},$$

with μ' denoting the dynamic viscosity.

Viewing the similarity in terms of usual assumptions, one may thus expect a single relation to apply to all geometrically similar vortex tubes:

$$\eta_h = \eta_h(\mu, \pi_h, \mathrm{Re}).$$

Test results pertaining to vortex tubes are sometimes represented in terms of such relations.

According to similarity theory, the dimensionless relations for all geometrically similar tubes must remain invariant as long as the adiabatic exponent k for the operating medium remains the same along with the criterial numbers Eu, Ma, Re defined from the identicity conditions. The slight variation in k due to changes in the state variables of superheated steam is negligible. Moreover, the condition Eu = idem and Ma = idem can be replaced (at a constant temperature T_1^0) by the condition P_1^0/P_h^0 = idem and P_1^0/P_c^0 = idem.

The test data indicate, however, that a relation of the $\pi_h = \eta_h(\mu, \pi_h, \mathrm{Re})$ kind is ambiguous and thus unsuitable for a generalization of test results pertaining to vortex tubes.

NOTATION

D	is the diameter of a vortex tube at the nozzle section;
k	is the isentropic exponent;
P_1^0	is the steam pressure at the tube entrance;
P_c^0	is the steam pressure at the cold end of a tube;
P_h^0	is the steam pressure at the hot end of a tube;
i	is the enthalpy of steam at the tube entrance;
$\Delta i_c = i - i_c$	is the decrease in enthalpy at the cold end of a tube;
μ	is the weight fraction of cold steam;

$\pi_h = P_1^0/P_h^0$	is the expansion ratio in the stream of hot steam;
$\pi_c = P_1^0/P_c^0$	is the expansion ratio in the stream of cold steam;
$Eu = P/\rho W^2$	is the Euler number;
$Ma = W/\rho$	is the Maievskii number;
W	is the stream velocity;
a	is the local velocity of sound;
ρ	is the density;
$\eta_h = \Delta i_h/\Delta i_s$	is the efficiency of a vortex tube;
Δi_h	is the increase in enthalpy at the hot end of a tube;
Δi_s	is the change in enthalpy corresponding to an isentropic expansion from P_1^0 to P_h^0.

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HEAT TRANSFER DURING BOILING OF CRYOGENIC LIQUIDS UNDER NATURAL CONVECTION

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UDC 536.423.1:546.21

The authors studied pool boiling of liquid nitrogen over the temperature range from −195 to −168°C and under pressures from 0.098 to 1.08 MN/m^2 at a horizontal tubular heater made of grade 1Kh18N9T steel.

Visual observations of nitrogen boiling have revealed four different process modes depending on the thermal flux density:

a) pure convection from 200 to 1900 W/m^2;

b) bubble boiling (isolated bubbles) from 1950 to 7700 W/m^2;

c) bubble boiling (merged bubbles — nucleation of vapor risers) from 8000 to 30,000 W/m^2;

d) developed bubble boiling (developed vapor risers) above 30,000 W/m^2.

The test data pertaining to the boiling of liquid nitrogen fit the following empirical relation (in the International System of units) with a maximum error of ±5%:

$$\alpha = 2.16\cdot 10^{-2} p^{0.38} q^{0.75}, \tag{1}$$

where α denotes the heat transfer coefficient, p denotes the pressure, and q denotes the thermal flux density.

Well known dimensionless relations for the rate of heat transfer during the boiling of nitrogen yield results which deviate appreciably from the test data. The closest agreement is obtained with the Borishanskii–Minchenko formula, within ±30% for $q < 30{,}000$ W/m^2 and within ±20% for $q > 30{,}000$ W/m^2.

The coefficient of heat transfer during the boiling of nitrogen or oxygen [2] is described by a relation which contains the dimensionless Borishanskii–Minchenko variables:

$$\frac{\alpha}{\lambda}\left[\frac{\sigma}{g(\rho_L-\rho_V)}\right]^{1/2} = 3.5\cdot 10^{-4}\left\{\frac{q}{a\rho_V L}\times\left[\frac{\sigma}{g(\rho_L-\rho_V)}\right]^{1/2}\right\}^{0.75}\left\{\frac{P}{\sigma}\left[\frac{\sigma}{g(\rho_L-\rho_V)}\right]^{1/2}\right\}^{0.75}, \tag{2}$$

with the thermal conductivity λ, the coefficient of surface tension σ, the acceleration due to gravity g, the thermal diffusivity a, the latent heat of evaporation L, the density of liquid and vapor ρ_L and ρ_V respectively, and with α, P, q defined earlier.

The maximum deviation of test data from relation (2) was ±15%.

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MASS TRANSFER DURING EXTRACTION OF A SOLID FROM A SEMIIFINITELY LARGE MEDIUM

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This article deals with the kinetics of extracting a solid substance from a semiinfinitely large porous medium under conditions where no insoluble solid or gas is formed at the same time, and a rather simple method is proposed here for determining the diffusivity. If the extract is a crystal hydrate with a mass fraction of the salt equal to ε, then the material balance in the extract from such a semiinfinitely large medium yields the following equation:

$$\frac{dl}{d\tau}[u_T+\psi m_p\Delta c]-m_{pk}\psi\frac{d}{d\tau}\int_0^l (c-c_1)\,dx=-D_K\delta\left(\frac{dc}{dx}\right)_{x=l}, \tag{1}$$

where

$$\psi=\frac{\rho_1-\alpha c_1}{\varepsilon\rho_1-c_1},\qquad \delta=\frac{\rho_1-c_1}{\varepsilon\rho_1-c_1}. \tag{2}$$

The integral equation (1) is used here for solving the mass transfer problem, which requires that the concentration function c be approximated by an equation which would satisfy the appropriate boundary conditions [1, 2]: $c = c_1$ at $x = 1$ and $c = c_s$ at $x = 0$. One such equation could be

$$\frac{c-c_1}{c_s-c_1}=1-\frac{x}{l}. \tag{3}$$

Inserting this into Eq. (1) and subsequent transformations yield

$$l=\eta\sqrt{\tau}. \tag{4}$$

Equation (4), expressed in terms of the function $l = f(\sqrt{\tau})$, represents a straight line with the slope η:

$$\eta=\sqrt{\frac{2\delta D_K}{\frac{u_T}{\Delta c}+\psi(m_p-0.5\,m_{pk})}}, \tag{5}$$

from which the diffusivity can be easily determined.

In order to determine D_K, it is necessary to set up an experiment so that the functional relation between l and τ can be established. As the test object the authors used a synthetic model of a porous body consisting of $CuSO_4 \cdot 5H_2O$ and an inert material (polystyrene) immersed in a solution of sulfuric acid. The magnitude of l was determined with the aid of a readout microscope. The value of η could be determined as $\tan\beta$, with β denoting the slope angle of the $l = f(\tau)$ lines.

The values of diffusivity were determined under the following conditions: $m_p = 0$, $c_1 = 0$, $\Delta c = c_s$, $\psi = 1/\varepsilon$, $\delta = 1/\varepsilon$. We found that $\varepsilon = 0.64$ for $CuSO_4 \cdot 5H_2O$. The values of diffusivity at temperatures of

20, 30, 40, and 50°C were found to be $0.226 \cdot 10^{-9}$, $0.48 \cdot 10^{-9}$, $0.662 \cdot 10^{-9}$, $0.974 \cdot 10^{-9}$ m^2/sec respectively.

A comparison between these values with those of natural diffusivity at the same temperatures indicates a rather close agreement. Thus, for example, at 20°C we have $D_K = 0.226 \cdot 10^{-9}$ m^2/sec and $D = 0.38 \cdot 10^{-9}$ m^2/sec.

NOTATION

u_T	is the mass of extracted substance per unit volume of the porous medium;
ε	is the mass fraction of salt in the crystal hydrate;
m_p	is the porosity of the medium before extraction;
m_{pk}	is the porosity of the medium after extraction;
c	is the instantaneous concentration in the solution;
c_1	is the concentration in the pool region of the solution;
c_s	is the saturation concentration;
ρ	is the instantaneous density of the solution;
ρ_1	is the density of the solution in the pool region;
D_K	is the effective diffusivity;
α	is the constant in Eq. (2);
τ	is the time;
l	is the depth of the extraction zone.

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MASS TRANSFER DURING EXTRACTION OF A SOLID FROM A SEMIINFINITELY LARGE MEDIUM COMPLICATED DUE TO CHEMICAL REACTIONS

E. M. Semenishin, G. A. Aksel'rud, and S. V. Limarenko

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This article deals with the kinetics of extracting a solid substance from a semiinfinitely large porous medium when a fast chemical reaction produces a soluble solid phase.

The extracted material in this study was malachite $CuCO_3 \cdot Cu(OH)_2$ distributed in the pores of an inert material (polystyrene), and a solution of sulfuric acid as the solvent.

In the first stage of the process, a strong reaction at the malachite surface produces gaseous CO_2 and solid $CuSO_4 \cdot 5H_2O$, the latter shielding the malachite particles and impeding an intensive continuation of the process. Owing to the poor solubility of copper vitriol in sulfuric acid, its diffusion into the pool region of the solution proceeds slowly. Sulfuric acid reaches the malachite surface only as $CuSO_4 \cdot 5H_2O$ withdraws. The process of mass transfer is thus constrained by its slowest stage, namely by the diffusion of copper vitriol into the ambient solution.

The presence of a protective $CuSO_4 \cdot 5H_2O$ film has been established by combining extraction and a periodic rinsing of the specimens with water, also by an x-ray structural examination of the reaction products.

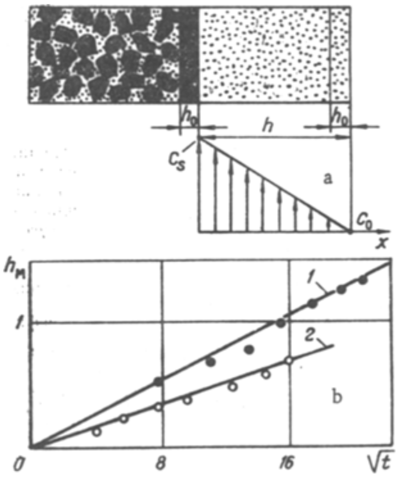

Fig. 1. Displacement of the interphase contact surface h_M as a function of $\sqrt{t}$: (a) schematic diagram of extraction of a solid substance; (b) extraction kinetics: curve 1) extraction of copper vitriol, curve 2) extraction of malachite.

In order to establish the kinetic laws governing the process of malachite extraction, the authors consider a model shown in Fig. 1a. The concentration of copper vitriol on the interphase contact surface is equal to the saturation concentration c_s. It decreases along the x-axis down to c_0. On the basis of Fick's law and of mass balance with regard to the extracted material, the authors have derived the equation

$$\sqrt{h^2-h_0^2}=\eta\sqrt{t}, \tag{1}$$

where

$$\eta=\sqrt{\frac{2Dc_s}{\rho}}. \tag{2}$$

Equation (1), expressed in terms of the function $\sqrt{h^2-h_0^2}=f(\sqrt{t})$, represents a straight line with the slope angle η.

Equation (1) can be reduced to

$$h_M=\eta\sqrt{t}. \tag{3}$$

A comparison between (1) and (3) indicates that the effect of a fast chemical reaction is to modify the extraction depth profile h. According to (1), h_M should be proportional to $\sqrt{t}$. In Fig. 1 we note this proportionality, which characterizes a simple extraction process (curve 2).

A smaller process motive force, resulting from a reduced saturation concentration of copper vitriol in the acid solution, explains why curve 2 is not as steep as curve 1 (Fig. 1b).

The concentration of acid has a significant effect on the kinetics of interaction between malachite and sulfuric acid. The experiment has shown that most favorable process conditions are realized at a 6% concentration of acid.

NOTATION

c_s is the saturation concentration;
c_0 is the concentration in the pool region of the solution;
D is the diffusivity;
ρ is the density of copper vitriol distribution;
t is the time;
h is the depth of the extraction zone;
h_0 is the zone of extracted material lost as a result of a chemical reaction.

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AN ANALYTICAL STUDY CONCERNING THE EFFECT OF THE VELOCITY OF THE DRYING AGENT ON THE RATE OF PEAT DEHYDRATION

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UDC 66.047

The effect of the velocity of the drying agent on the dehydration of peat in pneumogaseous desiccators is analyzed on the basis of a model set up here. The peat particles are regarded as existing in a state of stationary suspension in space. The velocity of a fictitious gas stream is in this case equal to the velocity of the drying agent and of the peat particles in a real process. Equations of material balance and of heat balance are also set up for this model.

An analysis of these equations shows that, as the air flow in such a pneumogaseous desiccator stabilizes, moisture is removed from the surface of peat particles at a rate which depends on the feed velocity of wet material and remains independent of the velocity of the drying agent.

For determining the velocity of peat particles in various desiccator zones, the authors propose a formula by which they have established that within the zone where particles are accelerated the desiccation rate increases with increasing velocity of the drying agent. The velocity of the drying agent gradually ceases completely to have an effect on the desiccation rate while the particles attain their full steady-state velocity.

An analysis of these relations also shows that the effect of particle size on the desiccation rate during the acceleration period becomes continuously stronger, up to some constant level.

The authors conclude, on the basis of this study, that peat can be desiccated at a faster rate, if the relative velocity of the drying agent is increased within the acceleration zone (by twisting the stream transversely to the direction of flow) and if heat is injected into the desiccation zone during the final stage of the process.

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THERMOPHYSICAL PROPERTIES AND THE STRUCTURE OF FILLED POLYFORMALDEHYDE AND FORMALDEHYDE—DIOXOLAN COPOLYMER

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UDC 536.21

The thermophysical properties of filled polymers during crystallization are largely determined by the structure of the matrix polymer at various levels of its supermolecular constitution.

A study was made concerning the effect of fillers on the thermophysical properties (thermal conductivity λ, thermal diffusivity a, and specific heat c_p) and on the structure, at various levels of its supermolecular constitution (degree of crystallinity, longitudinal and transverse dimensions of crystallites, and average size of spherolites), of grade A polyformaldehyde (PFA), stabilized 2% grade 54 polyamide with 2% diphenylamine, and of formaldehyde—1,3-dioxolan copolymer (CFD).

Aerosil of spherical particles 0.02 μm in diameter and with a specific surface of 175 m^2/g as well as acicular mullite single crystals 30-300 μm long and 3-8 μm wide of a $3Al_2O_3 \cdot 2SiO_2$ composition served as fillers.

The temperature-dependence of interplane distances d_{100} and d_{105} was established on the basis of fundamental interference fringes in a unit cell of PFA.

The changes in the thermophysical properties of PFA and CFD due to fillers added in small concentrations (1 to 5%) are explained by structural transformations which those fillers have effected in a block of the matrix polymer. An addition of such amounts of aerosil and mullite to these polymers raises the crystallinity level, increases both the longitudinal and the transverse dimensions of crystallites, reduces the size of spherolites, and produces a dimensionally more uniform supermolecular structure. All those effects combined, while causing the structural scatter of heat to diminish, increase the thermal conductivity λ and yield a specific heat c_p lower than that based on additive tests.

The changes in the thermophysical properties of PFA and CFD due to fillers added in large concentrations (10 to 30%) are explained by a combination of effects due to the thermophysical properties of the filler materials and due to further structural transformations occurring in a block of the matrix polymer.

The thermal diffusivity of all systems studied here decreases with rising temperature. Such an $a = f(T)$ trend can be explained in qualitative terms, if one considers that $a = \lambda / c_p \rho$. As the filler concentration increases, the thermal diffusivity a of all systems containing aerosil and mullite increases.

It has thus been demonstrated that, with fillers added in small amounts, the changes in the thermophysical properties of PFA and CFD are governed by structural transformations in the matrix polymer at various levels of its supermolecular constitution. With fillers added in large amounts, on the other hand, the effects of their thermophysical properties must also be taken into account.

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COMPOSITE DETERMINATION OF THE THERMOPHYSICAL PROPERTIES IN THIN LIQUID LAYERS

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UDC 536.223

An evaluation is made of the pulse method used for a composite determination of the thermophysical properties of dielectric liquids in thin layers.

The test liquid was poured into the plane-parallel gap between two semiinfinitely large bodies. On one boundary of this gap was placed a thin-film resistance probe. A rectangular current pulse was applied to it and the subsequent temperature rise of the sensing element was tracked (on the basis of the resistance change). The problem of transient heating is now formulated for this system and an exact analytical expression is derived for the probe temperature, which for a Fourier number $\mathrm{Fo} \leq 1$ becomes

$$T(\tau) = \frac{2q\sqrt{\tau}}{\sqrt{\pi}\,(\varepsilon_0 + \varepsilon_1)}\left[1 + \frac{2\sqrt{\pi}\,\varepsilon_0(\varepsilon_0 - \varepsilon_2)}{(\varepsilon_0 + \varepsilon_1)(\varepsilon_0 + \varepsilon_2)}\,\mathrm{ierfc}\,\frac{1}{\sqrt{\mathrm{Fo}}}\right].$$

With the aid of this formula and on the basis of temperature readings at two instants of time, the thermal activity and the thermal diffusivity of the test liquid are then calculated. A "compensation" method is described which makes it feasible to improve the accuracy of thermal diffusivity measurements. The method has been applied to two liquids: toluene and benzene. The results agree with known published data within 5%. The method is applicable to gap widths ranging from one to several hundred microns, with the required width of the test pulse varying accordingly from 5 μsec to 0.5 sec.

NOTATION

a	is the thermal diffusivity of the test liquid;
$\varepsilon_0, \varepsilon_1, \varepsilon_2$	are the thermal activities of the test substance and of the two semiinfinite bodies respectively;
q	is the thermal flux;
T	is the temperature;
τ	is the time;
L	is the layer thickness;
$\mathrm{Fo} = a\tau / L^2$	is the Fourier number.

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THERMAL CONDUCTIVITY OF ISOBUTYL ACETATE AT VARIOUS TEMPERATURES AND UNDER VARIOUS PRESSURES

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UDC 536.2:547.271

Results are shown of thermal conductivity measurements made on isobutyl ether of acetic acid over the 303.1-623.0°K temperature range and under pressures from 0.098 to 49 MN/m^2. The thermal conductivity of isobutyl acetate was measured with a cylindrical double calorimeter in the regular heating mode [1]. The thickness of the test layer of liquid and vapor was 0.47, 0.365, and 0.85 mm. The temperature drop across the liquid and the gaseous layer varied from 1.65 to 2.49°C.

Isobutyl acetate was additionally treated according to the procedure shown in [2] to a 99.97% purity, as indicated by a chromatographic analysis.

The thermal conductivity of isobutyl acetate was measured first. The results shown in Fig. 1 apply to temperatures up to 623°K and pressures up to 49 MN/m^2 in the liquid phase and in the vapor phase as well as near the saturation line. The error is estimated at 2.5%. Measurements near the saturation line were performed according to the procedure shown in [3]. For the points closest to the saturation line, $Gr \cdot Pr < 1000$. The results obtained by direct measurements near the saturation line were then compared with the results of thermal conductivity measurements on the basis of isotherms near the saturation line. The difference between both methods did not exceed 2-3%. One may conclude, therefore, that a temperature drop across the layer of test liquid (vapor) has no significant effect on the final result of thermal conductivity measurements near the saturation line.

Near the critical point the thermal conductivity λ increases anomalously. This, in the authors' view, is related to an energy transfer by separate molecules continuously building up and breaking down in the critical state.

According to measurements, the thermal conductivity of gaseous isobutyl acetate increases at a certain temperature (thermal pyrolysis [4]).

At temperatures within the 573-623°K range, the molecules of isobutyl acetate break down as follows:

$$C_4H_9 - COO - CH_3 \rightarrow C_4H_8 + CH_3COOH.$$

One molecule of isobutyl acetate yields a molecule of isobutylene (C_4H_6) and a molecule of acetic acid (CH_3COOH). In this way, the number of molecules participating in the heat transfer increases and so does then the thermal conductivity λ of the gas. The range associated with thermal pyrolysis is indicated in

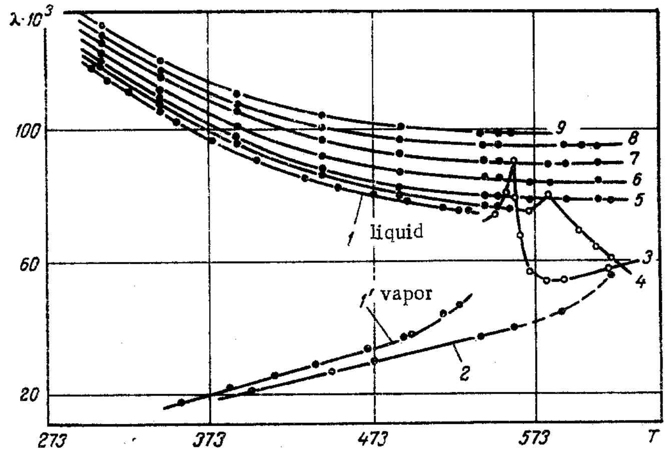

Fig. 1. Thermal conductivity λ (W/m·deg) of isobutyl acetate at various temperatures T (°K) and under various pressures: 1, 1') near the saturation line; 2) 0.098 MN/m^2; 3) 2.94 MN/m^2; 4) 4.9 MN/m^2; 5) 9.8 MN/m^2; 6) 19.6 MN/m^2; 7) 29.4 MN/m^2; 8) 39.2 MN/m^2; 9) 49.0 MN/m^2.

Fig. 1 by a dashed line. The anomalous increase in the thermal conductivity λ of the gas at high temperatures is very well explained by the Butler–Brokaw theory of dissociating gases [5].

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VISCOSITY OF CEMENT–SAND CONCRETE MIXES AND STRUCTURAL MORTARS

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UDC 691:539.55

The physical nature of colloidal cement paste in structural mortars and in concrete mixes has not been thoroughly enough explored to this day and is of considerable interest to the building industry where the improvement of technological processes is concerned.

This study on the subject has made it possible to establish how the dynamic viscosity varies with the size and the shape of colloidal particles, with the time of particle transition from one equilibrium state to another, with the activation energy, the temperature, and the external force on the mass. The following formula is to describe this relation:

$$\mu = \frac{1}{\left[1 + \frac{\left(\frac{A}{kT}\right)^2}{3!} + \frac{\left(\frac{A}{kT}\right)^4}{5!} + \ldots\right]} \frac{2kTt_0}{V} e^{\frac{\Delta U}{kT}},$$

where μ denotes the dynamic viscosity, k represents the Boltzmann constant, T denotes the temperature, t_0 denotes the time of particle transition from one equilibrium state to another, ΔU denotes the activation energy of a colloidal particle, A denotes the work of an external force through the path of a colloidal particle, and V denotes the volume of a colloidal particle.

The derivation of this formula is based on the diffusion theory of particle motion.

Measurements have yielded values for several physical parameters of colloidal paste in structural mortars and concrete mixes.

The test data indicate that, when the difference between actual and shearing stress and shear strength $(\tau - \tau_0)$ changes from 0.08 to 3.75 $\mathrm{kgf/m^2}$ due to a change in the transport velocity from 0.1 to 0.3 m/sec and a change in the velocity gradient from 2 to 5 $\mathrm{sec^{-1}}$, then the dynamic viscosity μ of the mix change from 0.4 to 0.75 $\mathrm{kgf \cdot sec/m^2}$. The activation energy of colloidal particles increases correspondingly from $0.352 \cdot 10^{-20}$ to $0.5 \cdot 10^{-20}$ kgf·m.

The test results pertaining to the dynamic viscosity μ as a function of the water-to-cement ratio, of the cement (Portland) consumption rate, of the mix mobility, and of the transport velocity are shown in Fig. 1 in the form of nomograms.

From the mortar grade R_m and the cement grade R_c we calculate the ratio R_c/R_m, and from this we find the point on curve 1 whose abscissa indicates the water-to-cement ratio; a perpendicular line is then drawn to intersect one curve on the lower nomogram corresponding to the cement consumption rate.

OPTIMIZING THE FRACTIONAL COMPOSITION OF A LOOSE MATERIAL FOR CLOSEST PACKING

T. T. Gribnyak and L. G. Gur'ev

UDC 621.798-492.3

The problem of filling a volume with spherical bodies assumes some practical significance in the development of metal-ceramics, in terms of optimizing the powder mixture as well as in terms of producing the most efficient fractional composition of minute round grains in vibratory grinders. The geometrical method is generally used in theoretical analysis when an ideal packing arrangement is realized: the interstices between spheres of equal dimensions are filled with correspondingly smaller spheres. Such a packing is not feasible in practice, however, because the passages which join the interstitial cavities are still smaller in cross section. This is the cause of discrepancies between experimental and theoretical results. In this study the authors take an entirely different approach to the problem. The size distribution of spherical particles is hypothetically described by a polynomial with unknown coefficients. A system of equations for these coefficients is set up on the basis of a maximized earlier equation for the number of contacts between loose particles. It appears that the solution to this system are coefficients of a polynomial of the third degree with respect to the referred particle radius. After normalization, this polynomial yields a size distribution of loose particles, and the closest packing can be obtained here on the assumption that the packing density increases with the number of contacts. The distribution function thus derived is then used for calculating the fractional composition. The packing of a powder mixture on this basis is compared with an experimental mixture produced by V. V. Okhotnyi. The number of contacts in the designed mixture is larger by two order of magnitude.

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Abstract submitted December 12, 1972.

A METHOD OF DETERMINING THE TEMPERATURE FIELD OF A BOUNDED MEDIUM

N. M. Tsirel'man

UDC 536.2.01

Thermal processes in media whose properties are temperature-dependent can be represented in terms of displacements of constant-temperature θ fronts. Each θ = idem front travels through the medium with fictitiously constant thermophysical properties which change to another constant level for the next θ = idem front, etc. The path of any θ = idem front depends on some fictitious thermal diffusivity a_f accounting for the history of the material prior to the front passage as well as for the thermal state of the material after the front passage. The quantity $x/\sqrt{4a_0\tau}$, which appears as the argument in certain solutions to nonlinear problems in a half-space, makes it feasible to determine a_f by one method which is explained here on an example.

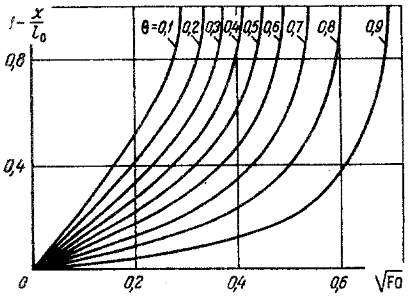

Fig. 1. Path of θ = idem fronts in an infinitely long cylinder with Bi = ∞.

The travel of θ = idem fronts through bounded media is to be represented as an initial displacement in the peripheral layers, in accordance with the laws applicable to a half-space where $a_f = a_f(\theta)$, and a subsequent penetration deeper into the region of already regular kinetics.

In view of this, the following procedure is proposed for determining the temperature fields of bounded media. From well known temperature relations of the thermophysical properties one finds the solution to the nonlinear equation of heat conduction a half-space and integrates this solution as the path of θ = idem fronts in x, $\sqrt{a_0\tau}$ coordinates. The overlapping of the θ-idem lines on the graph with the initial path segments of the θ = idem fronts, plotted in $(1-x/l_0)$, $\sqrt{\mathrm{Fo}} = a_0\tau/l_0^2$ coordinates on the basis of well known

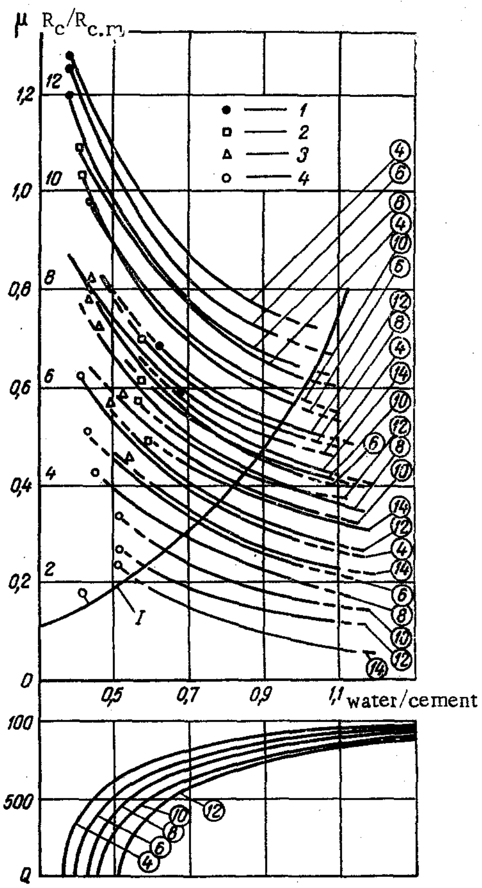

Fig. 1. Nomograms for determining μ of a cement–sand mortar and concrete mix; encircled numbers indicate the dip of a "Stroitsnil" funnel (cm): 1) v = 0.5 m/sec; 2) v = 0.3 m /sec; 3) v = 0.2 m/sec; 4) v = 0.1 m /sec; I) variation of $R_c/R_{c.m}$ with varying water-to-cement ratio in sandy concretes and mortars; Q denotes the consumption rate of cement (kg weight per 1 m^3 of mix).

The cement consumption rate and the water-to-cement ratio determine the mix mobility according to a "Stroitsnil" funnel. From the thus determined mobility, water-to-cement ratio, and known transport velocity we then obtain the value of the dynamic viscosity.

Example. Given are the mortar grade R_m-75, the grade of Portland cement "300," also the mix mobility equal to 6 cm (according to a "Stroitsnil" funnel) and the transport velocity v = 0.5 m/sec. Find the dynamic viscosity μ.

We find the ratio $R_c/R_m = 300/75 = 4$.

On the axis of ordinates we lay off this value of R_c/R_m, then draw a line parallel to the axis of abscissas till it intersects with curve I.

Through the intersection point we draw a line perpendicular to the axis of abscissas. This line intersects several curves in the upper and in the lower nomogram at points corresponding to a cement quality (water-to-cement ratio) 0.8, and will yield the cement consumption rate as well as its dynamic viscosity.

A cement consumption rate of 400 kg (weight) per 1 m^3 of mix is found on the lower nomogram from the intersection of this line with the mobility curve at 6 cm, and the dynamic viscosity is found on the upper nomogram from the intersection of the straight line with the viscosity curve at v = 0.5 m/sec and with the mobility curve at 6 cm.

In this way, the dynamic viscosity of the mix under the given conditions is found to be u = 0.77 kgf $\cdot$sec/m^2.

The nomogram indicates the mobility of a mix according to a "Stroitsnil" funnel.

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Abstract submitted November 1, 1972.

solutions to linear problems of heat conduction in bounded media, singles out and defines a family of θ = idem curves representing the temperature field, for the nonlinear case now.

An example is shown where this procedure is applied in combination with the well known solution to the linear problem concerning the temperature field of an infinitely long cylinder, and the necessary graphical interpretation is given on a diagram.

NOTATION

θ	is the dimensionless instantaneous temperature;
τ	is the time;
$\mathrm{Fo} = a_0\tau/l_0^2$	is the Fourier number;
a_f	is the fictitious thermal diffusivity at $\theta \neq 0$;
a_0	is the true thermal diffusivity at $\theta = 0$;
x	is the distance bounding the half-space;
l_0	is the characteristic dimension;
x/l_0	is the dimensionless space coordinate.

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Abstract submitted November 3, 1972.

EQUIVALENT THERMAL RESISTANCE TO CONVECTIVE HEAT TRANSFER DURING TRANSIENT HEAT CONDUCTION

N. M. Tsirel'man

UDC 536.25

On the basis of the formulas derived here for the velocity of constant-temperature θ fronts under boundary conditions of the first or the third kind, it is easy to show that the finite dimensionless travel time Fo of θ = idem fronts from the point in a body where their motion begins to follow a regular heating mode to the center of the body x/l_0 is

$$-\frac{1}{\mu_1^2}\ln\cos\mu_1\frac{x}{l_0}\Bigg|_{\frac{x}{l_0}}^{0};\quad -\frac{1}{\mu_1^2}\ln J_0\left(\mu_1\frac{x}{l_0}\right)\Bigg|_{\frac{x}{l_0}}^{0};\quad -\frac{1}{\mu_1^2}\ln\frac{\sin\mu_1\frac{x}{l_0}}{\frac{x}{l_0}}\Bigg|_{0}^{\frac{x}{l_0}}$$

respectively for a plate, a cylinder, and a sphere.

For boundary conditions of the first kind we have $u_1 = \pi/2$, 2.4048, π respectively and inserting these values into the formulas for Fo at $x/l_0 = 1$ yields Fo = ∞, corresponding to an infinite time of heating a body to $\theta \approx 1$ with a front which travels through the region of regular heating from almost the outer edge to the center.

For more general boundary conditions the third kind (where μ_1 is a function of the Biot number), we obtain Fo = ∞ at some distance $x/l_0 > 1$ determined from the condition Fo = ∞ for an arbitrary value μ_1 and equal to

$$\frac{\pi}{2\mu_1};\quad \frac{2.4048}{\mu_1};\quad \frac{\pi}{\mu_1}$$

respectively for a plate, a cylinder, and a sphere.

Thus, the travel of a $\theta \approx 1$ front through a body with a characteristic dimension l_0 under boundary conditions of the third kind can be replaced by such a travel in a body with a characteristic dimension equal to

$$\frac{\pi l_0}{2\mu_1};\quad \frac{2.4048\, l_0}{\mu_1};\quad \frac{\pi l_0}{\mu_1}$$

respectively for a plate, a cylinder, and a sphere under boundary conditions of the first kind.

Calculations have shown that at a Biot number Bi ≈ 5 to ∞ such a change of boundary conditions does not distort the travel pattern of $\theta = 0.1$-1.0 fronts, while at a Biot number Bi = 0.004-1.0 it results in a permissible error in the determination of Fo only for $\theta = 0.9$-1.0, i.e., for the final stages of front travel processes.

An example illustrates the practical use of this proposed change of boundary conditions for the determination of temperature fields.

NOTATION

θ	is the dimensionless temperature;
l_0	is the characteristic dimension;
x/l_0	is the dimensionless space coordinate;
Fo	is the Fourier number;
μ_1	is the first root of the characteristic equation;
Bi	is the Biot number.

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THERMODYNAMICS OF IRREVERSIBLE PROCESSES FOR CONTINUOUS SYSTEMS WHEN EXTERNAL FORCE FIELD IS UNSTEADY

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UDC 530.162

The thermodynamics (phenomenological theory) of irreversible processes is studied for continuous systems when the external force field is unsteady, although its vectors do not change their orientation in space with time. The curl of the force field is equal to 0. Three kinds of force are considered: gravitational, centrifugal, and surface intermolecular macroforces. The region being considered is anisotropic.

The generalized currents and forces are sought by the method given in [1], i.e., the local energy balance is constructed and an expression is found for the production of entropy $\sigma[s]$ and for the dissipative function Ψ. The expression for the local potential energy balance is given in the following form:

$$\frac{\partial \dot{o}}{\partial t} = -\frac{\partial \Phi_i\,[0]}{\partial x_i} - \sum_{\gamma} (F_\gamma)_i\, \rho_\gamma \frac{dx_i}{dt} - \sum_{\gamma} x_i \rho_\gamma \frac{(\partial F_\gamma)_i}{\partial t}.$$

To find $\sigma[s]$ the generalized equation of Gibbs is taken in the following form [2].

$$T d\overset{+}{s} = d\overset{+}{u} - \frac{1}{\rho}\tau_{ij} d\varepsilon_{ij} - \sum_{\gamma} \overset{+}{\mu}_\gamma d\overset{+}{N}_\gamma - \sum_{\gamma} \overset{+}{N}_\gamma d_t \overset{+}{e}{}'_\gamma \quad (i,\ j = 1,\ 2,\ 3),$$

$$\overset{+}{e}{}'_\gamma = \frac{\tau'_{ij}\alpha_{ij}}{\rho_\gamma}, \quad \alpha_{ij} = \frac{V_0 \delta\varepsilon_{ij}}{\delta V} = \text{const},$$

$$\sum_{\gamma} x_i \rho_\gamma \frac{(\partial F_\gamma)_i}{\partial t} = \sum_{\gamma} \rho_\gamma \frac{\partial \overset{+}{e}{}'_\gamma}{\partial t}.$$

Finally the generalized currents are found: v_Ω is the rate of the chemical reactions, $I^*_{\gamma i}$ is the mass transport, I_{q_i} the heat transport, $P_{ij} = P_{ji}$ are the frictional forces and the generalized forces analogous to them: A_Ω (affinity), $(X^*_\gamma)_i$, $(X^*_q)_i$, $X_{ij} = X_{ji} = -1/2(\partial w_i/\partial x_j + \partial w_j/\partial x_i)$.

A fundamentally new expression is obtained for

$$(X_\gamma)_i = \left[(F_\gamma)_i - \frac{\partial\,[(F_\gamma)_j x_j]}{(\Delta_\gamma)_i\,\partial t} - \left(\frac{\partial \overset{+}{\mu}_\gamma}{\partial x_i}\right)_T\right].$$

The phenomenological laws are formulated:

$$v_\Omega = \sum_{\Omega=1}^{R} a_{\Omega\Omega'}A_{\Omega'} + \sum_{\Omega=1}^{R} a_{\Omega q}(X_q)_i\delta_i + \sum_{\Omega=1}^{R}\sum_{\gamma=2}^{N} a_{\Omega\gamma}(X_\gamma^*)_i\,\delta_i + \sum_{\Omega=1}^{R} a_{\Omega p}X_{ij}\beta_{ij}, \tag{1}$$

$$(I_\gamma^*)_i = \sum_{\gamma=2}^{N}\sum_{\Omega=1}^{R} a_{\gamma\Omega}A_\Omega\,\delta_i + \sum_{\gamma=2}^{N} a_{\gamma q}(X_q)_i + \sum_{\gamma=2}^{N} a_{\gamma\gamma'}(X_{\gamma'}^*)_i + \sum_{\gamma=2}^{N} a_{\gamma p}X_{ij}\delta_j, \tag{2}$$

$$(I_q)_i = \sum_{\Omega=1}^{R} a_{q\Omega}\,A_\Omega\delta_i + a_{qq}(X_q)_i + \sum_{\gamma=2}^{N} a_{q\gamma}(X_\gamma^*)_i + a_{qp}X_{ij}\delta_j, \tag{3}$$

$$P_{ij} = P_{ji} = \sum_{\Omega=1}^{R} a_{p\Omega}A_\Omega\beta_{ij} + a_{pq}(X_q)_i\delta_j + \sum_{\gamma=2}^{N} a_{p\gamma}(X_\gamma^*)_i\,\delta_j - \eta\left(\frac{\partial w_i}{\partial x_j} + \frac{\partial w_j}{\partial x_i}\right) + \left(\frac{2}{3}\eta - \zeta\right)\delta_{ij}\frac{\partial w_\beta}{\partial x_\beta}. \tag{4}$$

NOTATION

$\dot{o}, \dot{s}, \dot{u}$	are the local specific potential energy (J/m^3), entropy, and internal energy;
$\partial\Phi_i[0]/\partial x_i$	is the so-called flow of potential energy;
$(F_\gamma)_i$	is the specific force acting on a particle of the γ type (N/kg);
ρ, ρ_γ	are the densities (at a point) of material and of particles of the γ type;
T	is the absolute temperature;
$\overset{+}{s}, \overset{+}{u}, N_\gamma$	are $\dot{s}/\rho$, $\dot{u}/\rho$, ρ_γ/ρ;
τ_{ij}	is the stress tensor;
ε_{ij}	is the deformation tensor;
τ'_{ij}	is the part of τ_{ij} produced by the existence of the external force field;
$\overset{+}{\mu}_\gamma$	is the specific chemical potential of material of particles of the γ type;
$a_{\Omega\Omega'}, a_{\gamma\gamma'}$	are the phenomenological coefficients;
δ_i, β_{ij}	are the unit dimensionless vector and tensor with matrices (111) and $\begin{pmatrix}111\\111\\111\end{pmatrix}$;
δ_{ij}	is the Kronecker delta;
η, ζ	are the coefficients of viscosity and second viscosity;
$(\Delta_\gamma)_i = (w_\gamma)_i - w_i$;	
w_i; $(w_\gamma)_i$	are the velocities of center of mass and of particles of the γ type;
V	is the volume;
V_0	is the initial volume.

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Abstract submitted November 3, 1972.

SOLUTION OF SPATIAL PROBLEM OF THERMAL CONDUCTION FOR SEMIINFINITE CYLINDER WITH MIXED BOUNDARY CONDITIONS

P. Z. Livshits

UDC 536.2

In the article a solution is constructed by the method proposed in [1, 2] for the equation of stationary thermal conduction for an unbounded cylinder satisfying the uniform mixed boundary conditions at $\rho = 1$:

$$T_n = 0,\ \zeta \geqslant 0;\ \frac{1}{\mathrm{Bi}}\frac{\partial T_n}{\partial \rho} + T_n = 0,\ \zeta < 0.$$

Two problems on the arbitrary heating of the end of a semifinite cylinder with uniform mixed boundary conditions ($l = c/a$) are examined with the help of the solutions constructed:

$$\rho = 1 : T_n = 0,\ 0 \leqslant \zeta \leqslant l;\ \frac{1}{\mathrm{Bi}}\frac{\partial T_n}{\partial \rho} + T_n = 0,\ \zeta < 0; \tag{1}$$

$$\zeta = l : T_n = f(\rho),\ 0 \leqslant \rho \leqslant 1;$$

$$\rho = 1 : T_n = 0,\ 0 \leqslant \zeta < \infty;\ \frac{1}{\mathrm{Bi}}\frac{\partial T_n}{\partial \rho} + T_n = 0,\ -l \leqslant \zeta < 0; \tag{2}$$

$$\zeta = -l : T_n = f(\rho),\ 0 \leqslant \rho \leqslant 1.$$

The problem of the uniform (in the axial direction) heating of part of the end surface of a semifinite cylinder is solved:

$$\rho = 1 : T_n = T_{n0},\ 0 \leqslant \zeta \leqslant l;\ \frac{1}{\mathrm{Bi}}\frac{\partial T_n}{\partial \rho} + T_n = 0,\ -\infty < \zeta < 0;$$

$$\zeta = l : \frac{\partial T_n}{\partial \zeta} = 0,\ 0 \leqslant \rho \leqslant 1;$$

$$T_n(\rho, \zeta) = T_{n0}\rho^n + 2\sum_{s=1}^{\infty} X_s \frac{J_n(\mu_s \rho)}{\mu_s} \operatorname{ch} \mu_s (l - \zeta) \quad (0 \leqslant \zeta \leqslant l). \tag{3}$$

A solution is found for the problem of the arbitrary heating of a section of the surface of an unbounded cylinder in the case of heat exchange with a medium on the remaining part of the end surface.

In all the problems examined the conditions at the end surface are satisfied exactly. The coefficients in the series for the uniform solutions are determined from normal (according to Poincaré and Koch [3]) systems of algebraic equations of the type

$$\sum_{k=1}^{\infty} a_{sk} X_k = d_s,\quad s = 1,\ 2,\ \ldots;\ a_{sk} = \begin{cases} 1 + c_{kk} & (s = k) \\ c_{sk} & (s \neq k). \end{cases}$$

The asymptotic equations obtained make it possible to find the intensity of radial heat flow at the end surface near dividing line of the boundary conditions.

NOTATION

$\rho = r/a$; $\zeta = z/a$	are the dimensionless coordinates (a is the radius of the cylinder);
$T_n(\rho, \zeta) \exp(in\varphi)$	is a particular solution of the equation $\nabla^2 T = 0$;
$\mu_s \equiv \mu_{ns}$	are the positive nulls of the Bessel functions $J_n(\mu)$($n = 0, 1, 2, \ldots$);
$ah = \mathrm{Bi}$	is the Biot's criterion (h is the relative heat transfer coefficient).

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Abstract submitted November 21, 1972.

NONSTATIONARY MIXED PROBLEM OF THERMAL CONDUCTIVITY FOR THE HALF-SPACE

A. P. Popova

UDC 536.25

The following axisymmetric problem is considered. Heat transfer to a medium at the temperature $T_0(r)$ is maintained in conformity with Newton's law in a part of the half-space $z > 0$ which constitutes a circle with the radius R. The temperature assigned outside the circle over the surface of the half-space varies according to the harmonic law $T = v_0(r)\exp(i\omega t)$.

The Hankel integral transform is used to reduce the above problem to a dual integral equation and then to the following Fredholm integral equation by means of Noble's method [1]:

$$\psi(x) + \frac{\varkappa + \nu}{\pi} \int_0^1 [k(|x-\tau|) - k(x+\tau)]\,\psi(\tau)\,d\tau = f(x), \quad x < 1,$$

$$k(z) = \ln\frac{1}{|z|} + \frac{1}{\varkappa+\nu}\int_0^\infty \frac{\sqrt{t^2+\nu^2} - (t+\nu)}{t}\cos zt\,dt, \quad \nu = R\sqrt{\frac{i\omega}{a}} = R\mu.$$

Here, $\varkappa$ is the Biot number and a is the thermal diffusivity. The right-hand side is expressed integrally in terms of the assigned functions v_0 and T_0.

The solution of the integral equation, obtained by means of the method used in [2], has the following form:

$$\psi(x) = f(x) - \frac{2(\varkappa+\nu)}{\pi}\sum_{m=1}^{\infty} a_m\psi_m \sin\frac{m\pi x}{2}, \quad a_m = c_m + d_m(\varkappa+\nu)^{-1},$$

$$c_m = \int_0^2 \ln\frac{1}{|z|}\cos\frac{m\pi z}{2}\,dz = \begin{cases} \sqrt{2}\,(1-\ln 2), & m = 0,\\ 2(m\pi)^{-1}\,Si(m\pi), & m > 0,\end{cases}$$

$$d_m = \frac{\pi}{2}\left[\frac{\sqrt{\gamma_m^2+\nu^2}-\nu}{\gamma_m} - 1\right] - \frac{\nu}{\gamma_m}\,Si(m\pi) - (-1)^m\frac{\nu^2}{\gamma_m^2}\int_0^1 \frac{\sqrt{1-\tau^2}\exp(-2\nu\tau)\,d\tau}{1+\nu^2\gamma_m^{-2}\tau^2}, \quad \gamma_m = \frac{m\pi}{2}.$$

The coefficients ψ_m are found from the regular infinite system

$$\psi_n + \frac{2(\varkappa+\nu)}{\pi}\sum_{m=1}^{\infty} a_m b_{nm}\psi_m = f_n,$$

$$b_{nm} = b_{n-m} - b_{n+m}, \quad b_k = \frac{2\sin\frac{k\pi}{2}}{k\pi}, \quad f_n = \int \sin\frac{n\pi x}{2}\,f(x)\,dx,$$

which is solved by using the reduction method.

The numerical realization of the method is given for the case $\mu = 0$, $v_0(r) = 0$, which was investigated in [3] by using the approximate method. For the above particular case, the temperature at the half-space boundary for $r < R$ is given by the expression

$$u(R,0) = u^*(\rho,0) = \frac{2\varkappa T_0}{\pi}\sqrt{1-\rho^2}\left[1 - \frac{4\varkappa}{\pi}\sum_{k=0}^{\infty}\frac{(2k+3/2)\;2k!!\;A_k^{(N)}}{(2k+1)!!}\cdot\frac{P_{2k+1}\left(\sqrt{1-\rho^2}\right)}{\sqrt{1-\rho^2}}\right].$$

Here T_0 = const, $P_n(x)$ are Legendre polynomials, and $A_k^{(N)}$ are expressed in terms of the coefficients ψ_m.

Comparison between the numerical results and the results obtained by means of the approximate method in [3] shows good agreement for Biot numbers 0.1 and 1.0.

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Abstract submitted November 3, 1972.

CALCULATION OF THE MOTION OF DROPS IN FRONT OF A LATTICE OF CYLINDERS

A. V. Alekseev, V. D. Goryachev,
and B. G. Efremov

UDC 621.928.96

The article deals with the problem of calculating the efficiency of the capture of small drops of water by a lattice of cylinders placed across the flow.

The equations of motion of the drops were integrated by the Bless method on a "Nairi" computer. It was assumed that the velocity of motion of a drop far from the lattice is equal to the velocity of the carrying stream of gas.

The problem was solved for two types of cylinder lattices: for a single-row lattice of cylinders with a relative transverse pitch of σ_1 and for a "three-dimensional" lattice consisting of three rows of cylinders arranged in checkerboard fashion. The geometry of the lattice was determined by the relative transverse pitch σ_1 and the relative longitudinal pitch σ_2.

In order to calculate the velocities of the gas flow in the case of a single-row lattice of cylinders, we used the analytic solution obtained in [1]. In the case of the multi-row lattice we used the EGDA method. The determining criterion used in the calculations was the number

$$K = \frac{H}{\mathrm{Stk}},$$

where $H = 12.5/24\sqrt{u \sim d_{dr}/\nu_g}$ and Stk = the Stokes number.

As the calculations showed, reducing σ_1 to 1.11 increases the efficiency of capture of the drops by one cylinder of the lattice by 15% over that of an isolated cylinder.

Determination of the critical conditions of drop capture by a lattice of cylinders, i.e., the conditions at which the drops of water are no longer captured by the lattice, by electronic computers is made more difficult by the sharp increase in the machine time required for the calculations. The use of the method described in [2] made it possible to calculate the critical Stokes number determining such conditions. We obtained the following formula for the critical Stokes number:

$$\mathrm{Stk}_{\mathrm{cr}} = \frac{0.25}{p^3 \operatorname{sh} 2p \left[\frac{m_1}{(\operatorname{sh}^2 p + \sin^2 p\lambda)^2} + \frac{m_2}{\operatorname{sh}^4 p}\right]}, \tag{2}$$

where p, m_1, m_2, and λ are dimensionless parameters of the lattice.

In order to determine the effect of the geometry of the multi-row lattice on the efficiency of drop capture, we carried out calculations for four variants of the lattices: $\sigma_1 = 2$, $\sigma_2 = 2$; $\sigma_1 = 2$, $\sigma_2 = 1$; $\sigma_1 = 2$, $\sigma_2 = 0.75$; and $\sigma_1 = 1.5$, $\sigma_2 = 1$. The calculations showed that the effect of the transverse pitch on the efficiency of capture of particles by the lattice is greater than the effect of the longitudinal pitch.

Since the calculations neglected the effect of flow separations and turbulence beyond the first and second rows of the lattice, the results obtained should be considered as a first approximation which will have to be corrected experimentally.

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IONOMETRIC INVESTIGATIONS OF THE EFFECT OF PROTON IRRADIATION ON PURE AND ALLOYED BISMUTH

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Using the method of ionometry the authors obtained information on imperfections of the crystal lattice in single crystals of pure bismuth grade Bi 000 and its alloys with tin of three concentrations, 0.2, 0.6, and 1 at. %, upon their irradiation by 100 keV protons. The same specimens were subjected to irradiation by an integral flux of $5 \cdot 10^{15}$ cm^{-2} and $1.0 \cdot 10^{16}$ cm^{-2}. After each of the irradiations the backscatter spectra were recorded, then the specimens were annealed, and once again investigated by the same method. Annealing was done for 4 h at 120°C.

Calculation of the defects formed under the effect of irradiation was done with the use of the orientation relationship of the magnitude of the Rutherford scattering yield during channelling of a 1.4 MeV analyzing beam of helium ions by means of a slightly modified method used in [1]. The axial spectra with orientation of the analyzing beam in the direction of axis ⟨0009⟩ were recorded by means of a semiconductor detector and 256-channel analyzer.

Concentration relationships of the number of displaced atoms were obtained for the first and second irradiations. With an increase of lead in the impurity material there was a tendency toward a decrease of the number of defects formed on irradiation, which evidently can be related with the addition of the lead impurity, leading to strengthening of the bismuth lattice [2].

Attention is called to the fact that the number of defects per proton after the second irradiation with a large integral flux is less than after the first irradiation. It is suggested that after the first irradiation and subsequent annealing the main effects are those of imperfections in the irradiated layer of the monocrystalline structure of the pure and alloyed bismuth specimens and the formation of clusters of defects that are stable up to the recrystallization temperature, analogous to that found for graphite after irradiation and annealing [3]. The second irradiation leads mainly to the formation of the usual radiation defects, a considerable proportion of which is captured by traps, and to a small increase of the number of stable defects. Annealing of defects, increasing with increase of the impurity content during the second annealing, can serve as confirmation of the assumption of the presence of traps of simple defects in the irradiated layer after the second effect.

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